TRINITY COLLEGE

IN NOMINE DOMINI

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (94 Marks)**

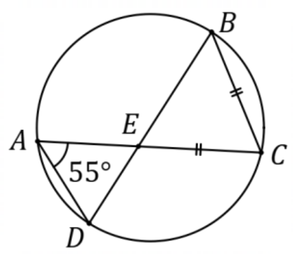
This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(8 marks)**

(a) The diagram shows the chords AC and BD of a circle intersecting at E. $\angle DAE = 55°$ and $CB = CE$.

Determine the size of $\angle EBC$ and the size of $\angle ADE$.

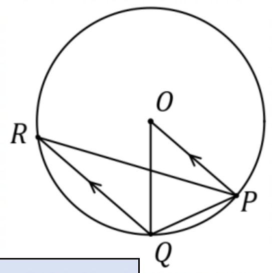


(2 marks)

Solution
$\angle EBC = \angle DAC = 55°$
$\angle ADE = \angle ACB = 180° - 2\angle DBC = 70°$
Specific behaviours
✓ correct $\angle EBC$ ✓ correct $\angle ADE$

(b) The diagram shows a circle, centre O, and points P, Q and R that lie on it. RQ is parallel to OP.

Determine the size of $\angle OQP$ and the size of $\angle RQP$ when $\angle OPR = 18°$.

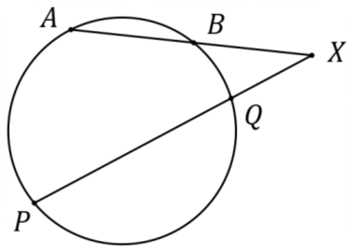


(3 marks)

Solution
$\angle PRQ = \angle OPR = 18°$ $\angle POQ = 2 \times \angle PRQ = 2 \times 18° = 36°$ $\angle OQP = (180° - \angle POQ) \div 2 = (180° - 36°) \div 2 = 72°$ $\angle RQP = 180° - \angle OPQ = 180° - 72° = 108°$ $(\angle OPQ = \angle OQP)$
Specific behaviours
✓ correct $\angle POQ$ ✓ correct $\angle OQP$ ✓ correct $\angle RQP$

(c) In the diagram, secants AB and PQ of a circle intersect at X.

Determine the length AB when $PQ = 8$ cm, $XQ = 4$ cm and $XB = 4.8$ cm.



(3 marks)

Solution
Intersecting secants: $XB \times XA = XQ \times XP$. $XP = 4 + 8 = 12, \quad AB = x, \quad XA = x + 4.8$ $4.8(4.8 + x) = 4 \times 12$ $x = 5.2$ Hence $AB = 5.2$ cm.
Specific behaviours
✓ indicates correct segment lengths ✓ equation using intersecting secant theorem ✓ correct length

Question 9 **(7 marks)**

Let $\underset{\sim}{a} = \begin{pmatrix} k \\ 2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 3 \\ k+1 \end{pmatrix}$. Determine

(a) $\underset{\sim}{b} - \underset{\sim}{a}$ when $k = 1$. (1 mark)

Solution
$\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$
Specific behaviours
✓ correct vector

(b) the angle in degrees between the directions of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ when $k = -9$. (1 mark)

Solution
Using CAS, angle is $123°$.
Specific behaviours
✓ correct angle

(c) the value(s) of k for which $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular. (2 marks)

Solution
$\underset{\sim}{a} \cdot \underset{\sim}{b} = 0 \Rightarrow 3k + 2(k+1) = 0$ $5k + 2 = 0, \quad k = -\frac{2}{5}$
Specific behaviours
✓ indicates scalar product is 0 ✓ correct value of k

(d) the value(s) of k for which $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel. (3 marks)

Solution
$\underset{\sim}{a} = \mu \underset{\sim}{b} \Rightarrow \mu = \frac{k}{3} = \frac{2}{k+1}$ $k^2 + k - 6 = 0$ $(k-2)(k+3) = 0$ $k = 2, \quad k = -3$
Specific behaviours
✓ forms equation   ✓ one value of k ✓ all correct values of k

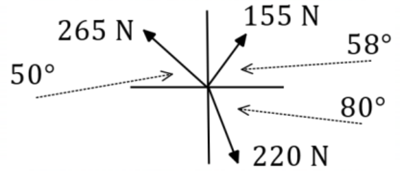
Question 10 **(10 marks)**

Three coplanar forces act on a small body. One force of 265 N acts on a bearing of 320°, another of 155 N acts on a bearing of 032° and another of 220 N acts on a bearing of 170°.

Let the perpendicular unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ act in an easterly and northerly direction respectively, and in this question give coefficients correct to 0.01.

(a) Sketch a diagram of the three forces acting on the small body. (1 mark)

Solution



Specific behaviours

✓ forces in correct quadrants, angles not required

(b) Express the force of 155 N using the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$. (2 marks)

Solution

Force of 155 N acts at an angle of $90° - 32° = 58°$ to positive x-axis:

$$\underset{\sim}{f_1} = 155\begin{pmatrix}\cos(58°)\\ \sin(58°)\end{pmatrix} = 82.14\underset{\sim}{i} + 131.45\underset{\sim}{j}$$

Note that correct to 0.01 is for guidance only.

Specific behaviours

✓ indicates correct angle with x-axis
✓ correct coefficients in any component form: ordered pair, column vector, etc.

(c) Determine the resultant of the three forces, stating its magnitude and the bearing in which it acts. (4 marks)

Solution

$$\underset{\sim}{f_2} = 265\begin{pmatrix}\cos(90° - 320°)\\ \sin(90° - 320°)\end{pmatrix} = \begin{pmatrix}-170.34\\ 203.00\end{pmatrix}$$

$$\underset{\sim}{f_3} = 220\begin{pmatrix}\cos(90° - 170°)\\ \sin(90° - 170°)\end{pmatrix} = \begin{pmatrix}38.20\\ -216.66\end{pmatrix}$$

$$\underset{\sim}{R} = \begin{pmatrix}82.14\\ 131.45\end{pmatrix} + \begin{pmatrix}-170.34\\ 203.00\end{pmatrix} + \begin{pmatrix}38.20\\ -216.66\end{pmatrix} = \begin{pmatrix}-50\\ 117.79\end{pmatrix}$$

$$\left|\underset{\sim}{R}\right| = \sqrt{50^2 + 117.79^2} = 127.96, \qquad \theta = \tan^{-1}\left(\frac{117.79}{-50}\right) = 113°$$

Resultant is 128 N acting on a bearing of $90° - 113° + 360° = 337°$.

Specific behaviours

✓ converts all forces into component form
✓ obtains resultant in component form
✓ correct magnitude of resultant
✓ correct bearing of resultant

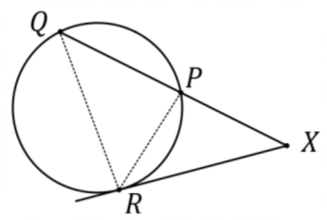
(d) Can the three forces be brought to a state of equilibrium by changing the direction in which the force of 155 N acts? Justify your answer. (3 marks)

Solution
No. The resultant of the other two forces is: $\underset{\sim}{R} = \begin{pmatrix} -170.34 \\ 203.00 \end{pmatrix} + \begin{pmatrix} 38.20 \\ -216.66 \end{pmatrix} = \begin{pmatrix} -132.14 \\ -13.66 \end{pmatrix}$ $\left\lvert \underset{\sim}{R} \right\rvert = \sqrt{132.14^2 + 13.66^2} = 132.84 \text{ N}$ For equilibrium, the force of 155 N would have to be equal in magnitude and opposite in direction to $\underset{\sim}{R}$ and since $\left\lvert \underset{\sim}{R} \right\rvert = 132.84$ N, this can never be the case.
Specific behaviours
✓ calculates resultant of other two forces ✓ calculates magnitude of resultant ✓ states no, with reasoning

Question 10 **(8 marks)**

A secant from point X meets a circle first at P and then at Q, and a tangent from X touches the same circle at R.

(a) Sketch a diagram to show the circle, secant, tangent and four points. (1 mark)

Solution

Specific behaviours
✓ correct diagram

(b) Prove that $\Delta XQR \sim \Delta XPR$ and hence show that $XP \times XQ = XR^2$. (4 marks)

Solution

$\angle QXR = \angle RXP$ (Common angle)

$\angle XQR = \angle XRP$ (Angles in opposite segments)

Hence $\Delta XQR \sim \Delta XPR$ as corresponding angles are equal.

Using ratios of corresponding sides:

$$\frac{XP}{XR} = \frac{XR}{XQ} \Rightarrow XP \times XQ = XR^2$$

Specific behaviours

✓ shows congruency of one pair of angles, with reasoning
✓ shows congruency of second pair of angles, with reasoning
✓ establishes similarity, with reasoning
✓ uses ratio of sides to obtain required result

(c) Determine the exact length of PQ when $XR = 14$ cm and $4 \times PQ = 3 \times XP$. (3 marks)

Solution

Let $PQ = x$. Then:

$$XP = \frac{4x}{3}, \qquad XQ = x + \frac{4x}{3} = \frac{7x}{3}$$

$$\frac{4x}{3} \times \frac{7x}{3} = 14^2$$

$$x^2 = \frac{196 \times 9}{28}$$

$$x = 3\sqrt{7}\text{ cm}$$

Specific behaviours

✓ expressions for XP and XQ
✓ uses result from (b) to form equation
✓ correct length in exact form

Question 12 **(7 marks)**

(a) An unordered selection of 4 dishes must be chosen from 5 hot and 9 cold dishes. Determine the number of ways the selection can be made if

(i) there are no restrictions. (1 mark)

Solution
$\binom{14}{4} = 1001$ ways.
Specific behaviours
✓ correct number of ways

(ii) there must be at least one of each type and at least as many cold dishes as hot. (2 marks)

Solution
$(c, h) = (3, 1)$ or $(2, 2)$ $\binom{9}{3}\binom{5}{1} + \binom{9}{2}\binom{5}{2} = 84 \times 5 + 36 \times 10 = 420 + 360 = 780$ ways.
Specific behaviours
✓ indicates correct method   ✓ correct number of ways

(b) Consider the following identity associated with Pascal's triangle:

$$\binom{n}{r+s}\binom{r+s}{r} = \binom{n}{r}\binom{n-r}{s}, \qquad \{n, r, s\} \in \mathbb{Z}^+, n \geq r + s.$$

(i) Show that the identity is true when $n = 8, r = 5$ and $s = 2$. (1 mark)

Solution
$LHS = \binom{8}{7}\binom{7}{5} = 8 \times 21 = 168, \qquad RHS = \binom{8}{5}\binom{3}{2} = 56 \times 3 = 168$ Hence $LHS = RHS$ and identity is true.
Specific behaviours
✓ shows substitution and evaluates for both sides of identity

(ii) Prove the identity is true. (3 marks)

Solution

$$\begin{aligned} LHS &= \binom{n}{r+s}\binom{r+s}{r} \\ &= \frac{n!}{(r+s)!\,(n-r-s)!} \times \frac{(r+s)!}{r!\,s!} \\ &= \frac{n!}{r!} \times \frac{1}{s!\,(n-r-s)!} \times \frac{(n-r)!}{(n-r)!} \\ &= \frac{n!}{r!\,(n-r)!} \times \frac{(n-r)!}{s!\,(n-r-s)!} \\ &= \binom{n}{r}\binom{n-r}{s} \\ &= RHS \end{aligned}$$

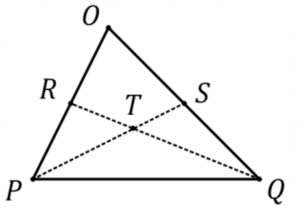
Specific behaviours

✓ correctly expands LHS and/or RHS using factorial fractions
✓ eliminates $(r+s)!$ terms and introduces $(n-r)!$ terms
✓ arranges ready to express fractions in ${}^nC_r$ form, and does so

Question 13 **(8 marks)**

In ΔOPQ, R and S are the midpoints of sides OP and OQ respectively. PS intersects QR at T.

(a) Sketch a diagram to show this information. (1 mark)

Solution

Specific behaviours
✓ correctly labelled diagram

Let $\underset{\sim}{p} = \overrightarrow{OP},\ \underset{\sim}{q} = \overrightarrow{OQ},\ \overrightarrow{PT} = \lambda \times \overrightarrow{PS}$ and $\overrightarrow{QT} = \mu \times \overrightarrow{QR}$.

(b) Express $\overrightarrow{OT}$ in terms of $\underset{\sim}{p}, \underset{\sim}{q}$ and λ. (2 marks)

Solution
$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT}$ $= \overrightarrow{OP} + \lambda\overrightarrow{PS}$ $= \underset{\sim}{p} + \lambda\left(-\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right)$
Specific behaviours
✓ correct vector for $\overrightarrow{PS}$ ✓ correct expression

(c) Express $\overrightarrow{OT}$ in terms of $\underset{\sim}{p}, \underset{\sim}{q}$ and μ. (1 mark)

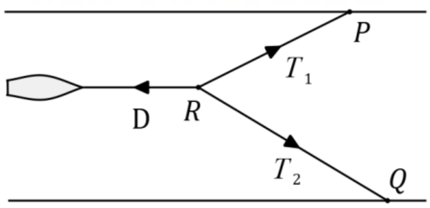
Solution
$\overrightarrow{OT} = \overrightarrow{OQ} + \overrightarrow{QT}$ $= \overrightarrow{OQ} + \lambda\overrightarrow{QR}$ $= \underset{\sim}{q} + \mu\left(-\underset{\sim}{q} + \frac{1}{2}\underset{\sim}{p}\right)$
Specific behaviours
✓ correct expression

(d) Prove that $\overrightarrow{OP} + \overrightarrow{OQ} = 3 \times \overrightarrow{OT}$. (4 marks)

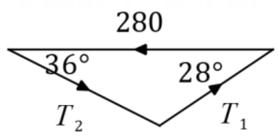
Solution
From parts (b) and (c) above we can equate both vectors for $\overrightarrow{OT}$: $\underset{\sim}{p} + \lambda\left(-\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right) = \underset{\sim}{q} + \mu\left(-\underset{\sim}{q} + \frac{1}{2}\underset{\sim}{p}\right)$ Considering $\underset{\sim}{p}$ and $\underset{\sim}{q}$ coefficients: $1 - \lambda = \frac{\mu}{2}, \qquad \frac{\lambda}{2} = 1 - \mu$ Solving simultaneously, $\lambda = \mu = \frac{2}{3}$. Hence $3\,\overrightarrow{OT} = 3\left(\underset{\sim}{p} + \frac{2}{3}\left(-\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right)\right) = 3\underset{\sim}{p} - 2\underset{\sim}{p} + \underset{\sim}{q} = \underset{\sim}{p} + \underset{\sim}{q} = \overrightarrow{OP} + \overrightarrow{OQ}$
Specific behaviours
✓ equates vectors for $\overrightarrow{OT}$ ✓ forms equations using coefficients ✓ solves for at least one unknown ✓ completes proof

Question 14 **(8 marks)**

A steady current of 0.6 m/s flows westwards between the parallel banks of a river that are 165 m apart. The diagram below shows a small boat is moored by ropes PR and QR that make angles of $28°$ and $36°$ respectively with the banks of the river.



(a) Determine T_1 and T_2, the tensions in each rope, when the drag force D caused by the water passing the boat is 280 N. (4 marks)

Solution
 Using vector triangle: $$\frac{T_1}{\sin 36°} = \frac{T_2}{\sin 28°} = \frac{280}{\sin 116°}$$ $T_1 = 183$ N, $T_2 = 146$ N
Specific behaviours
✓ forms vector triangle using tensions and drag ✓ indicates correct use of sine rule ✓ one correct tension ✓ second correct tension

(b) Another small boat leaves Q at a speed of 5 m/s and heads slightly upstream, steering a course that makes an angle of $80°$ with the bank of the river. Determine how far the boat is from P when it reaches the opposite bank, given that P is 20 m downstream from Q. (4 marks)

Solution
Time to cross river: $165 \div (5 \sin 80°) = 33.5$ s. Eastward speed of boat: $5 \cos 80° - 0.6 = 0.268$ m/s. Eastward displacement: $33.5 \times 0.268 = 9.0$ m and so the boat will arrive at a point $20 + 9 = 29$ m from P.
Specific behaviours
✓ indicates speed perpendicular to riverbank ✓ obtains time to cross river ✓ indicates velocity parallel to riverbank ✓ correct distance from P

Question 15 **(8 marks)**

A charity is selling raffle tickets. The tickets are left over from a previous event and are numbered consecutively from 168 to 500 inclusive.

(a) Determine how many of the tickets have a number that is

(i) a multiple of 8. (2 marks)

Solution
$\lfloor 500 \div 8 \rfloor - \lfloor 167 \div 8 \rfloor = 62 - 20 = 42$ tickets.
Specific behaviours
✓ multiples between 1 and 500 ✓ subtracts multiples < 168 to obtain correct number

(ii) a multiple of 8 or a multiple of 14. (3 marks)

Solution
Multiples of 14: $$\lfloor 500 \div 14 \rfloor - \lfloor 167 \div 14 \rfloor = 35 - 11 = 24$$ Multiples of 56 (i.e., multiples of both using LCM of 8 and 14): $$\lfloor 500 \div 56 \rfloor - \lfloor 167 \div 56 \rfloor = 8 - 2 = 6$$ Using the inclusion-exclusion principle: $n = 42 + 24 - 6 = 60$ tickets.
Specific behaviours
✓ multiples of 14 in range ✓ multiples of 56 in range ✓ correctly uses inclusion-exclusion principle to obtain correct number

(iii) a multiple of 8 or a multiple of 14 but not a multiple of both. (1 mark)

Solution
$n = 60 - 6 = 54$ tickets.
Specific behaviours
✓ correct number

Once all the tickets have been sold, they will be placed in a large barrel, thoroughly mixed and drawn from the barrel at random and without replacement.

(b) After how many draws can you be certain that at least three of the tickets taken from the barrel have numbers that start with the same digit? Justify your answer. (2 marks)

Solution
5 pigeon-holes (labelled $1, 2, 3, 4, 5$) and so to be certain that at least one is filled with at least 3 pigeons, select $5 \times 2 + 1 = 11$ pigeons. You can be certain after 11 draws.
Specific behaviours
✓ correct number ✓ justifies answer

Question 16 **(7 marks)**

Six cubes, identical apart from their colour, are to be placed one on top of another to form a tower. Determine the number of different towers that can be made using:

(a) seven different coloured cubes. (1 mark)

Solution
${}^7P_6 = 5040$
Specific behaviours
✓ correct number of towers

(b) a red, a yellow and four blue cubes. (1 mark)

Solution
$\dfrac{6!}{4!} = 30$
Specific behaviours
✓ correct number of towers

(c) a green, two red and three pink cubes. (1 mark)

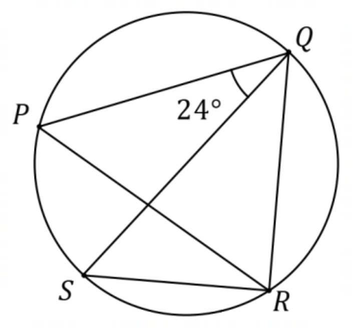
Solution
$\dfrac{6!}{3!\,2!} = 60$
Specific behaviours
✓ correct number of towers

(d) one yellow, one red, one blue, one purple, two black and two green cubes. (4 marks)

Solution
A. All different coloured cubes, arrange.
$n_A = 6! = 720$
B. Choose 2 same colour, choose 4 different, arrange.
$n_B = {}^2C_1 \times {}^5C_4 \times \dfrac{6!}{2!} = 2 \times 5 \times 360 = 3600$
C. Choose 2 same colour pairs, choose 2 different, arrange.
$n_C = {}^2C_2 \times {}^4C_2 \times \dfrac{6!}{2!\,2!} = 1 \times 6 \times 180 = 1080$
Total of $720 + 3600 + 1080 = 5400$ different towers.
Specific behaviours
✓ correctly splits into mutually exclusive cases
✓ one correct number of arrangements
✓ second correct number of arrangements
✓ correct total number of arrangements

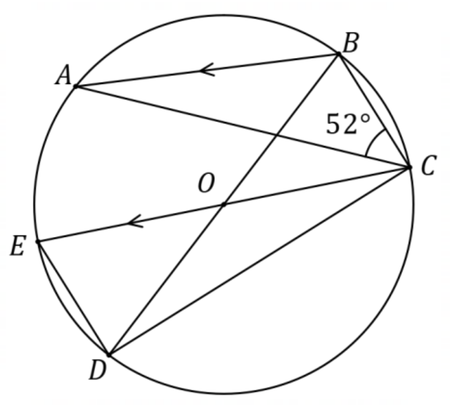
Question 17 **(6 marks)**

(a) In the diagram, points P, Q, R and S lie on a circle so that QS is a diameter and $PQ = PR$. Determine the size of $\angle RQS$ when $\angle PQS = 24°$. (3 marks)



Solution
$\angle PRS = \angle PQS = 24°$ (Angles on same arc) $\angle PRQ = 90° - 24° = 66°$ (Angle in semicircle) ΔPQR is isosceles, so $\angle PQR = \angle PRQ$. $\angle RQS = \angle PQR - \angle PQS$ $= 66° - 24°$ $= 42°$
Specific behaviours
✓ obtains $\angle PRS$ ✓ obtains $\angle PRQ$ ✓ correct $\angle RQS$

(b) In the diagram, points A, B, C, D and E lie on a circle with centre O. BD and CE are diameters, and AB is parallel to EC. Determine the size of $\angle OED$ when $\angle BCA = 52°$. (3 marks)

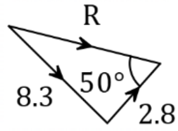


Solution
Angle in semicircle and angles on same arc: $\angle ABD = \angle ACD = 90° - 52° = 38°$ Let $\angle OED = \angle OBC = \angle OCB = x$. Sum of cointerior angles: $\angle ABC + \angle BCO = 180°$ $38° + x + x = 180°$ $x = \angle OED = 71°$
Specific behaviours
✓ obtains $\angle ABD$ ✓ forms equation using cointerior angles ✓ correct $\angle OED$

Question 18 **(9 marks)**

Water in a large river estuary flows at a constant 2.8 km/h on a bearing $070°$. At $1{:}45$ pm, a fishing boat in the estuary has just dropped a crab pot at A and heads off towards B to drop its last one, 1.47 km away on a bearing of $120°$. The fishing boat travels at a constant 8.3 km/h.

(a) Sketch a diagram to represent the sum of the water and boat velocities as it moves directly from A to B, including the angle between their resultant and the water velocity. (2 marks)

Solution	
Angle between resultant and water: $\theta = 120° - 70° = 50°$	
Specific behaviours	
✓ correct angle in triangle ✓ correctly shows sum of vectors	

(b) Determine the bearing the boat should steer from A to B. (2 marks)

Solution
$\dfrac{\sin x}{2.8} = \dfrac{\sin 50°}{8.3} \Rightarrow x = 15°$ Bearing to steer is $120° + 15° = 135°$.
Specific behaviours
✓ indicates equation for angle ✓ correct angle and bearing

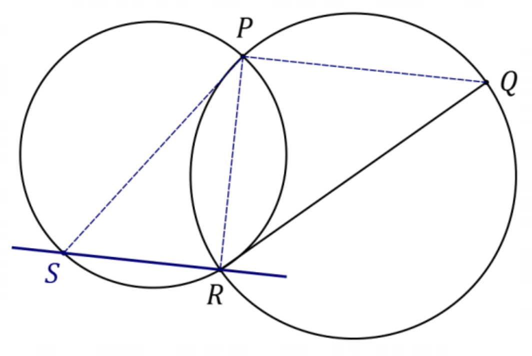
Once the boat reaches B it will spend 5 minutes dropping the last pot and then head off to an anchorage 2.2 km away from B on a bearing of $250°$.

(c) Determine the time at which the boat is expected to reach the anchorage. (5 marks)

Solution
Let magnitude of resultant be r and $180° - 50° - 15° = 115°$: $\dfrac{r}{\sin 115°} = \dfrac{8.3}{\sin 50°} \Rightarrow r = 9.82\ \text{km/h}$ Time from A to B is $1.47 \div 9.82 \times 60 = 9$ minutes. On way to anchorage, $250° - 070° = 180°$, so boat heading directly against current, and its resultant velocity is $8.3 - 2.8 = 5.5$ km/h. Time from B to anchorage is $2.2 \div 5.5 \times 60 = 24$ minutes and so arrival time: $1{:}45\ \text{pm} + 0{:}09 + 0{:}05 + 0{:}24 = 2{:}23$ pm.
Specific behaviours
✓ indicates equation for magnitude of resultant ✓ obtains magnitude of resultant ✓ correct time from A to B ✓ indicates correct resultant velocity from B to anchorage ✓ correct arrival time at anchorage

Question 19 **(8 marks)**

(a) The diagram shows two circles intersecting at P and R. RQ is the diameter of the larger circle.



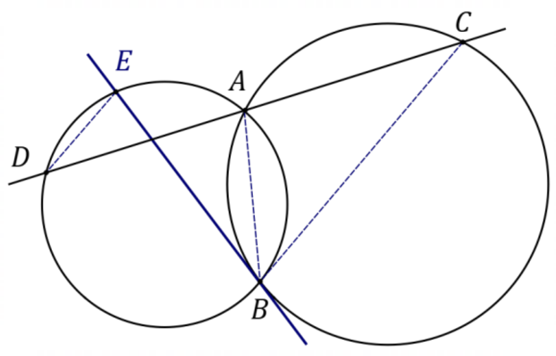
(i) Add to the diagram a straight line through R, parallel to PQ, that meets the smaller circle at S. (1 mark)

Solution
See diagram.
Specific behaviours
✓ correctly adds straight line

(ii) Prove that PS is a diameter of the smaller circle. (3 marks)

Solution
$\angle QPR = 90°$ (Angle in a semicircle) $\angle PRS = \angle QPR = 90°$ (Alternate angles, $PQ \parallel RS$) $\therefore$ PS is a diameter using converse of angle in semicircle.
Specific behaviours
✓ correctly uses angle in semicircle theorem ✓ correctly uses alternate angle theorem ✓ completes proof, with clear explanation throughout

(b) The diagram shows two circles intersecting at A and B. A straight line through A meets one circle at C and the other circle at D.



(i) Add to the diagram a tangent to circle ABC at B that meets circle ABD at E. (1 mark)

Solution
See diagram.
Specific behaviours
✓ correctly adds tangent

(ii) Prove that DE is parallel to BC. (3 marks)

Solution
$\angle BCA = \angle ABE$ (Alternate segment theorem) $\angle EDA = \angle EBA$ (Angles on same arc AE) $\therefore \angle EDA = \angle BCA$. Hence $ED \parallel BC$ as alternate angles $\angle EDA$ and $\angle BCA$ are equal.
Specific behaviours
✓ correctly uses alternate segment theorem ✓ correctly uses angles on same arc theorem ✓ shows $\angle EDA$ and $\angle BCA$ are equal and completes proof

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________